

9.3

Graph and Write Equations of Circles

Goal • Graph and write equations of circles.

Your Notes

VOCABULARY

Circle The set of all points (x, y) that are equidistant from a fixed point

Center The fixed point that is equidistant from all the points on a circle

Radius The distance r between the center and any point (x, y) on a circle

STANDARD EQUATION OF A CIRCLE WITH CENTER AT THE ORIGIN

The standard form of the equation of a circle with center at $(0, 0)$ and radius r is as follows:

$$x^2 + y^2 = r^2$$

Example 1 Graph an equation of a circle

Graph $y^2 = -x^2 + 16$. Identify the radius of the circle.

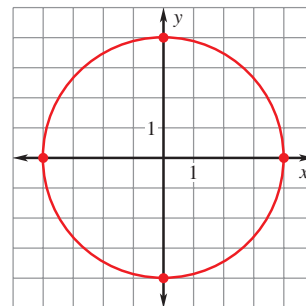
Solution

1. Rewrite the equation $y^2 = -x^2 + 16$ in standard form as $x^2 + y^2 = 16$.

2. Identify the center and radius.

From the equation, the graph is a circle centered at the origin with radius $r = \sqrt{16} = 4$.

3. Draw the circle. First plot several convenient points that are 4 units from the origin, such as $(0, 4)$, $(4, 0)$, $(0, -4)$, and $(-4, 0)$. Then draw the circle that passes through the points.

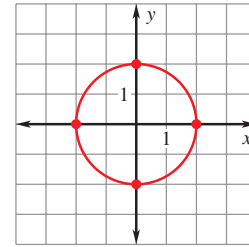


Your Notes

✓ **Checkpoint** Graph the equation. Identify the radius.

1. $x^2 = 4 - y^2$

$r = 2$



Example 2 Write an equation of a circle

The point $(-3, 4)$ lies on a circle whose center is the origin. Write the standard form of the equation of the circle.

The circle's radius r must be the distance between the center and $(-3, 4)$. Use the distance formula.

$$\begin{aligned} r &= \sqrt{(-3 - 0)^2 + (4 - 0)^2} \\ &= \sqrt{9 + 16} = \sqrt{25} = 5 \end{aligned}$$

Use the standard form with $r = 5$ to write an equation of the circle.

$$x^2 + y^2 = r^2 \quad \text{Standard form}$$

$$x^2 + y^2 = 5^2 \quad \text{Substitute for } r.$$

$$x^2 + y^2 = 25 \quad \text{Simplify.}$$

Example 3 Find a tangent line

Write an equation of the line tangent to the circle $x^2 + y^2 = 17$ at $(4, -1)$.

A line tangent to a circle and the radius to the point of tangency are perpendicular. The radius with endpoint $(4, -1)$ has slope $m = \frac{-1 - 0}{4 - 0} = -\frac{1}{4}$, so the slope of the tangent line at $(4, -1)$ is the negative reciprocal of $-\frac{1}{4}$, or 4 . An equation of the tangent line is as follows:

$$y + 1 = 4(x - 4) \quad \text{Point-slope form}$$

$$y = 4x - 17 \quad \text{Solve for } y.$$

Your Notes

Example 4 Write a circular model

Lighthouse The beam from Oak Island Lighthouse in North Carolina can be seen for up to 24 miles. You are 18 miles east and 9 miles south of the lighthouse. Can you see the lighthouse beam?

Solution

1. **Write** an inequality for the region lit by the beam.

This region is all the points that satisfy the following inequality: $x^2 + y^2 < \underline{24}^2$

2. **Substitute** the coordinates (18, 9) into the inequality.

$$x^2 + y^2 < \underline{24}^2 \quad \text{Inequality}$$

$$\underline{18^2 + 9^2} < \underline{24}^2 \quad \text{Substitute for } x \text{ and } y.$$

$$\underline{405} < \underline{576} \quad \text{The inequality is } \underline{\text{true}}.$$

You can see the lighthouse beam.

✓ Checkpoint Complete the following exercises.

2. Write the standard form of the equation of the circle with center at the origin that passes through the point (6, -3).

$$x^2 + y^2 = 45$$

3. Write an equation of the line tangent to the circle $x^2 + y^2 = 34$ at (-3, -5).

$$y = -\frac{3}{5}x - \frac{34}{5}$$

4. From Example 4, suppose you are 16 miles east and 19 miles south of the lighthouse. Can you see the lighthouse beam?

no

Homework